Operations with Functions

State the inverse of each function.

1. $f(x) = \frac{5}{2}x - 4$ $f^{-1}(x) = \frac{2}{5}x + \frac{8}{5} = \frac{2x+8}{5}$	2. $g(x) = \frac{9x+3}{5}$ $g^{-1}(x) = \frac{5}{9}x - \frac{1}{3} = \frac{5x-3}{9}$
3. $h(x) = 2x - \frac{1}{2}$ $h^{-1}(x) = \frac{1}{2}x + \frac{1}{4}$ $\frac{2x+1}{4}$	4. $f(x) = \frac{x + -4}{8}$ $f^{-1}(x) = 8x + 4$
5. $h(x) = \frac{5}{7}x - \frac{6}{7}$ $h^{-1}(x) = \frac{7}{5}x + \frac{6}{5}$	6. $g(x) = 2x + 1$ $g^{-1}(x) = \frac{1}{2}x - \frac{1}{2}$
7. $g(x) = \frac{-2x + -7}{6}$ $g^{-1}(x) = -3x - \frac{7}{2}$	8. $f(x) = 6x + 9$ $f^{-1}(x) = \frac{1}{6}x - \frac{3}{2}$
9. $h(x) = \frac{-15x + 20}{35}$ $h^{-1}(x) = \frac{-7}{3}x + \frac{4}{3}$	10. $h(x) = \frac{-8}{5}x + \frac{2}{5}$ $h^{-1}(x) = \frac{-5}{8}x + \frac{1}{4}$
11. $g(x) = \frac{7x + 1}{2}$ $g^{-1}(x) = \frac{2}{7}x - \frac{1}{7}$	12. $f(x) = \frac{5}{3}x + \frac{1}{3}$ $f^{-1}(x) = \frac{3}{5}x - \frac{1}{5}$
13. $f(x) = \frac{-8}{9}x + \frac{1}{9}$ $f^{-1}(x) = \frac{-9}{8}x + \frac{1}{8}$	14. $h(x) = \frac{3}{2}x + \frac{7}{4}$ $h^{-1}(x) = \frac{2}{3}x - \frac{7}{6}$
15. $f(x) = \frac{-28x + 20}{28}$ $f^{-1}(x) = -1x + \frac{5}{7}$	16. $h(x) = \frac{14x + -35}{42}$ $h^{-1}(x) = 3x + \frac{5}{2}$

Adding and Subtracting functions

1) Add or subtract the value of f(x), g(x), and/or h(x) normally.
2) make sure to combine line terms and watch your signs.
3) write your final answer.

$g(x) = 3x+5 \quad f(x) = 2x-3 \quad h(x) = \frac{3x}{2}$

$(f+g)(x) = (2x-3)+(3x+5)$

$(f+g)(x) = 5x+2$

$(g-h)(x) = (3x+5) - \left(\frac{3x}{2}\right)$

$= 3x+5 - \left(\frac{3x}{2}\right)$ $1.5x$

$= 1.5x+5$

$3x + 5 - \frac{3x}{2}$

$\frac{6x}{2} + \frac{10}{2} - \frac{3x}{2}$

$= \frac{3x}{2} + 5$

Multiplying functions
1) identify which functions you are multiplying
2) multiply normally - make sure to FOIL or use the distributive property when appropriate.
3) write your final answer

$f(x) = x+3 \quad h(x) = 3x-5 \quad g(x) = \frac{x}{2}$

$(fg)(x) = \left(\frac{x+3}{1}\right)\left(\frac{x}{2}\right) = \frac{x^2+3x}{2}$

$(fh)(x) = (x+3)(3x-5)$

$= 3x^2 - 5x + 9x - 15$

$= 3x^2 + 4x - 15$

Dividing functions
1) identify which functions you are dividing
2) write the dividend as the numerator
3) write the divisor as the denominator
4) determine any values that will make the denominator equal zero
5) write your final answer, including what x cannot equal

$f(x) = 2x - 4$
$g(x) = 5x - 3$
$h(x) = x^2$

$(f/g)(x) = \dfrac{2x-4}{5x-3}, x \neq \frac{3}{5}$

$5x - 3 = 0$
$+3 \quad +3$
$\frac{5x}{5} = \frac{3}{5}$
$x = \frac{3}{5}$

$(h/f)(x) = \dfrac{x^2}{2x-4}, x \neq 2$

$2x - 4 = 0$
$2x = 4$
$x = 2$

Composition of Functions
1) determine which function is the "inner" function
2) insert the "inner" function into the "outer" function every place where you see x
3) simplify
4) if necessary determine any values that would make the denominator zero

$f(x) = 2x - 3$
$g(x) = x + 1$
$h(x) = x^2$

$(f \circ g)(x) = 2(x+1) - 3$
outer inner
$= 2x + 2 - 3$
$= 2x - 1$

$(h \circ f)(x) = (2x-3)^2 = (2x-3)(2x-3)$
$= 4x^2 - 6x - 6x + 9 = 4x^2 - 12x + 9$

$(g \circ h)(x) = (x^2) + 1$

Composition of Functions and their Inverses

When you do the composition of a function and it's inverse, the result is x!

$$f(x) = 3x - 2$$

$$y = 3x - 2$$

$$x = 3y - 2$$

$$+2 \quad\quad +2$$

$$\frac{x+2}{3} = \frac{3y}{3}$$

$$y = \frac{x+2}{3}$$

$$f^{-1}(x) = \frac{x+2}{3}$$

$$(f \circ f^{-1})(x) =$$

$$= 3\left(\frac{x+2}{3}\right) - 2$$

$$= \frac{\cancel{3}}{1}\left(\frac{x+2}{\cancel{3}}\right) - 2$$

$$= x + 2 - 2$$

$$= x$$